Algebra Review Booklet

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The goal of this booklet is not to teach you how to do a particular math problem, but to refresh your memory on skills you once knew but may not have practiced for a while.

Some of this may seem simple, but it is worth practicing the easy stuff so that you are comfortable with the material and how it is presented in this booklet. Feel free to skip sections that you know well.

Real Numbers

Algebra generally deals with the set of numbers that we call “real numbers”. Real numbers include both positive and negative integers as well as those numbers in between, fractions or decimals, which are referred to as rational numbers. Real numbers also include irrational numbers, those numbers that never repeat and never end such as π, 0.010110111…, or $\sqrt{2}$.

Signed Numbers

When ***adding signed numbers*** (positive or negative), if the signs are the same, add the numbers and keep the sign.

For example:

$(-2) + (-3) = -5$

$(+4) + (+2) = +6$ or $4 + 2 = 6$

When you are adding signed numbers and the signs are not the same, start by ignoring the signs. Subtract the smaller number from the larger number and then keep the sign of the larger number for the answer.

For example:

$-7 + 3 \rightarrow 7 - 3 = 4$

Since 7 is larger and negative, the answer is negative, so

$-7 + 3 = -4$

When ***subtracting signed numbers***, rewrite the question as an addition and change the sign of the second number. Then, follow the rules above for addition.

For example:

$-4 - 5$ would be rewritten as $-4 + (-5)$;

the signs are the same, so add the numbers (4 and 5 = 9) and keep the negative sign

$-4 - 5 = -9$

For ***multiplication and division of signed numbers***, when the signs are the same, the answer is positive. When the signs are opposite, the answer is negative.

For example:

$4 \times 3 = 12$	$15 \div 5 = 3$
$-4 \times 3 = -12$	$-15 \div 5 = -3$
$-4 \times (-3) = 12$	$-15 \div (-5) = 3$
$4 \times (-3) = -12$	$15 \div (-5) = -3$

When entering calculations on a calculator, use the $\boxed{+/-}$ key to enter negative numbers.

The ***absolute value*** of a number is the number itself, without a sign. That is, the absolute value of −10 is 10. The absolute value of 10 is also 10.

$$|-10| = 10$$

$$|10| = 10$$

Practice – Real Numbers

1. $(-2) - 3$
2. $-2 + 0$
3. $4 - (-2)$
4. $5 \times (-2)$
5. $-16 \div (-1)$
6. $-8 \div 2$

(answers are on page 19)

Scientific Notation and Exponents

Scientific notation is a way to use powers of ten to write really large or really small numbers more efficiently. A number in scientific notation is written as a multiplication of a number between 1 and 10, and a power of ten.

When converting a large number from standard notation to scientific notation, the decimal must move to the left and the exponent is positive.

The mass of the Earth, 6,615,000,000,000,000,000,000 tons, is more easily written as 6.615×10^{21} tons.

The decimal must move to the right when it is a small number and then the exponent is negative.

A strand of human DNA is 0.00000000013 cm which can be written in scientific notation as 1.3×10^{-10} cm.

Exponents

When multiplying variables with exponents, we need to keep in mind what the exponents are actually telling us.

$y^2 = y \cdot y$

$y^3 = y \cdot y \cdot y$

so, $y^2 \cdot y^3 = (y \cdot y) \cdot (y \cdot y \cdot y)$ or y^5

The product rule for exponents: "When multiplying and the bases are the same, keep the base and add the exponents".

$$\frac{y^5}{y^2} = \frac{\cancel{y} \cdot \cancel{y} \cdot y \cdot y \cdot y}{\cancel{y} \cdot \cancel{y}} = y^3$$

The quotient rule for exponents: "When dividing and the bases are the same, keep the base and subtract the exponents".

Practice – Scientific Notation and Exponents

Convert to Scientific Notation:

7. 3,000,000 m/s

8. 0.000 000 024 m

Convert to Standard Notation:

9. 5.63×10^{7} tons

10. 2.234×10^{-9} Pa

Multiply or Divide:

11. $x^2y^2 \cdot x^3y$

12. $5x^4y^3z \div x^3y$

(answers are on page 19)

Evaluating Expressions

In order not to make an error when substituting values into an expression, use brackets around the substituted value.

To evaluate the expression: $4x - 3y$
when $x = -2$ and $y = -5$

Rewrite the expression and insert the values given for x and y. Follow the rules for order of operations (BEDMAS) to solve.

$$4(-2) - 3(-5)$$

$$-8 + 15$$

$$7$$

Practice – Evaluating Expressions

13. $x - 2$ when $x = 9$

14. $2a^2b + c(a + b)$ when $a = -1$, $b = 1$, $c = 2$

15. $\dfrac{\frac{1}{2}(x + y)}{(x - y)}$ when $x = 12$ and $y = 2$

(answers are on page 19)

Simplifying Expressions

Simplifying an expression means to combine like terms. "Like terms" must have exactly the same variable part including the exponents.

For example, $6x^2yz$ and $4xy^2z$ are NOT like terms.

To simplify the expression $5x^2y + 3y^2 - 3x^2y$ combine only those terms that are exactly alike. The terms that are alike are $5x^2y$ and $(-3x^2y)$, so the simplified expression is

$$\boxed{5x^2y} + 3y^2 \boxed{-3x^2y}$$

$$2x^2y + 3y^2$$

Practice – Simplifying Expressions

16. $2xy^2 + 4x - 3y^2 + (-3xy^2) - 3x$

17. $10abc - 3b^2 + 15ab + abc + 3ab + 3b^2$

(answers are on page 19)

Solving Equations

An equation is a number sentence that says that the expressions on each side of the equal sign represent the same number. When solving equations, the goal is to determine what value will make the equation true. We do this by getting the variable by itself on one side of the equation.

Solving by Addition or Subtraction

As long as the same amount is added or subtracted from both sides of the equation, the equation will remain equal. The choice of what to add or subtract is based on the goal of isolating the variable, the unknown.

$Q - 5 = 12$ (the opposite of subtraction is addition)

$Q - 5 + 5 = 12 + 5$ (add 5 to both sides to get Q alone)

$Q = 17$

Doing the opposite operation will result in the variable and zero. When a number is added to the variable, we need to subtract that number. When a number is being subtracted, we need to add.

$3 = Q + 2$ (the opposite of addition is subtraction)

$3 - 2 = Q + 2 - 2$ (subtract 2 from both sides to get Q alone)

$1 = Q$

Solving by Multiplication or Division

As long as the same amount is multiplied or divided on both sides of the equation, the equation will remain equal.

$4A = 24$ (the opposite of multiplication is division)

$\frac{4A}{4} = \frac{24}{4}$ (divide both sides by 4 to get A alone)

$A = 6$

Doing the opposite operation result in the variable times one. When a number is multiplied to the variable, we need to divide by that number. When the variable is being divided by a number, we need to multiply.

$\frac{y}{2} = 6$ (the opposite of division is multiplication)

$2(\frac{y}{2}) = 2(6)$ (multiply both sides by 2 to get *y* alone)

$y = 12$

You could choose to multiply both sides of the equation to clear fractions or decimals. Multiply by the lowest common denominator to clear fractions. To clear decimals, multiply by a multiple of 10.

$x + 0.5 = 1.6$ (multiply by 10)

$10(x + 0.5) = 10(1.6)$

$10x + 5 = 16$ (subtract 5 from both sides)

$10x = 11$ (divide by 10 to get x alone)

$x = 1.1$

Practice – Solving Equations

18. $N + 75 = 101$

19. $42 = R - 23$

20. $5m = 60$

21. $4x + 2 = 18$

22. $\frac{2}{3} + \frac{1}{4}t = \frac{1}{3}$

23. $6(2x - 5) - 12 = 15 + 3(x - 1)$

24. A rope 50 centimetres long is cut into three pieces. The first piece is twice as long as the second piece. The third piece is 2 centimetres longer than the second. What are the lengths of the three pieces?

(answers are on page 19)

Operations with Polynomials

Expressions can be made up of one or more terms. For example, the expression $2x^2 + 4x - 5$ is a polynomial with three terms. The three terms are: $2x^2$, $4x$, and -5. Note that the sign in front of a term belongs to that term. Because this polynomial has three terms, we could also call it a trinomial.

Just like real numbers, polynomials can be added, subtracted, multiplied, or divided. Begin by removing brackets, and simplify the expression by collecting like terms.

Adding and Subtracting Polynomial

The coefficients of the like terms are added or subtracted and the variable part stays the same.

$$(5x^2 + 3x - 2) + (3x^2 - 2x + 6)$$

$$8x^2 + x + 4$$

$$(5x^2 + 3x - 2) - (3x^2 - 2x + 6)$$

To subtract polynomials, rewrite the subtraction as an addition by multiplying the minus sign into the brackets. This will make the signs opposite on each of the terms in the second polynomial. Then collect like terms.

$$(5x^2 + 3x - 2) + (-3x^2 + 2x - 6)$$

$$2x^2 + 5x - 8$$

Multiplying Polynomials

To find an equivalent expression for the product of two monomials (polynomials with only one term), multiply the coefficients and then multiply the variables using the product rule for exponents.

$(3x)(4x) = 12x^2$

$(4x^2y)(2x^3y^2z) = 8x^5y^3z$

To multiply a monomial into a polynomial, multiply the monomial to EACH term in the polynomial. (We often call this distributing.)

$3x(4x^3y + 2x^2y - 2x^2 + 6y) \;=\; 12x^4y + 6x^3y - 6x^3 + 18xy$

To multiply two polynomials, multiply each term in the first polynomial to each term in the second one. Then, collect like terms.

$(3x^2 + 4)(2x^2 - 5) \;=\; 6x^4 - 15x^2 + 8x^2 - 20 \;=\; 6x^4 - 7x^2 - 20$

If you recognize some of the special cases when multiplying polynomials, such as difference of squares or trinomial squares, you can save a bit of time, but distributing terms always works.

Dividing Polynomials

Division of polynomials follows similar steps as division of real numbers. Divide the coefficients and then divide the variables using the quotient rule for exponents.

Division by a monomial is most easily done by reducing fractions.

$5b^2 - 20ab + 15a^3b \div 5b$

Can be written as

$$\frac{5b2 - 20ab + 15a3b}{5b} = \frac{5b^2}{5b} + \frac{-20ab}{5b} + \frac{15a^3b}{5b} = b - 4a + 3a^3$$

Practice – Operations with Polynomials

25. $(2x^2 + 4x + 3) + (4x^2 - 2x - 1)$
26. $(4x^3 - 2x + 3) - (3x^2 - x + 4)$
27. $2x^2z(3y^2z^2 + x)$
28. $(2x^2 + 4y)(3x^2 + y - 2z^3)$
29. $(8w^4 + 6w^3 - 12w) \div 2w$

(answers are on page 19)

Factoring

To factor something is to express it as a multiplication that is equivalent.

For example, $15 = 5 \cdot 3$ (5 and 3 are factors of 15)

$4x^3y = 2 \cdot 2 \cdot x \cdot x \cdot x \cdot y$ (2, x, and y are factors of $4x^3y$)

When factoring expressions, factor out the greatest common factor.

$5x^3y + 10x^2y^3 = 5x^2y(x + 2y^2)$

Using a strategy to factor expressions will save you time and trouble.

1. Look first for common factors. If there is one, factor out the largest common factor.

2. Then, count the number of terms.

   2 terms

   Check for a difference of squares ($A^2 - B^2$). Do not try to factor a sum of squares

   3 terms

   Check for a trinomial square ($A^2 + 2AB + B^2$).
   If not, try FOIL, trial and error, or the *ac*-method

   4 terms

   Try factoring by grouping

3. Always factor completely! If a factor with more than one term can still be factored, you should factor it.

4. Check your answer by multiplying.

To factor $5x^2 + 12xy + 7y^2$ using the *ac*-method, start by multiplying the first coefficient, (*a)* and the last coefficient (*c*) to get 35.

Then, find the pair of factors of 35 that adds to the middle coefficient, 12.

Use the pair to split the middle term into two terms.

$5x^2 + 12xy + 7y^2$	(split the middle term)
$(5x^2 + 7xy) + (5xy + 7y^2)$	(group)
$x(5x + 7y) + y(5x + 7y)$	(remove common factors)
$(5x + 7y)(x + y)$	(pull out the common factors)

Practice – Factoring

Factor completely:

30. $15x^2y - 25y^2$

31. $4x^2 - 16x + 16$

32. $25z^2 - 16$

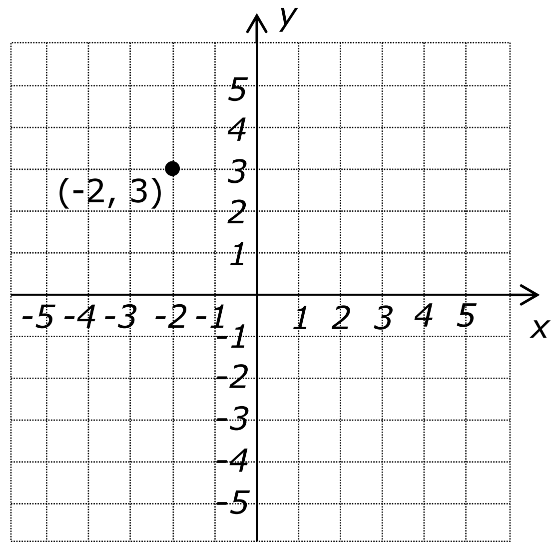
33. $x^2 + x - 12$

(answers are on page 19)

Graphing

In the Cartesian coordinate system, the first value of a coordinate pair is plotted by counting on the graph the appropriate number of units. For the first value, start from the origin (the centre) and count to the right when the value is positive and to the left when the value is negative. The second value is plotted by counting up from the origin for positive values and down for negative.

The ordered pair (–2, 3) would be plotted by counting two units to the left and three units up.



A graph of an equation is a drawing that represents all of its solutions. When an equation contains two variables, the solutions of the equation are ordered pairs. To determine whether a pair is a solution, we use the first number in each pair to replace the variable as they occur alphabetically.

The standard equation for a line is in the form $y = mx + b$ where m represents the slope of the line and b the coordinate where the line crosses the y axis (0, b).

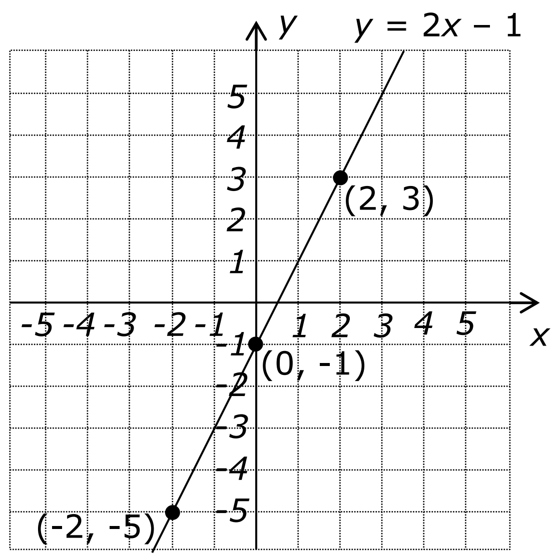
When the slope, m, is negative, the line will angle downward from left to right. The line will angle upward from left to right when the slope is positive. The larger the value of m, the steeper the slope.

To graph a linear equation

1. Select a value for one variable and substitute into the equation to calculate the corresponding value of the other variable. Form an ordered pair in alphabetical order.
2. Repeat step 1 to obtain at least two other ordered pairs.
3. Plot the ordered pairs and draw a straight line passing through the points.

To graph $y = 2x - 1$

x	y	(x,y)
-2	-5	$(-2, -5)$
0	-1	$(0, -1)$
2	3	$(2, 3)$



Any other point that the line passes through is also a solution to the equation.

The slope of this line, m, is equal to 2. This means the line will rise 2 units for each unit to the right.

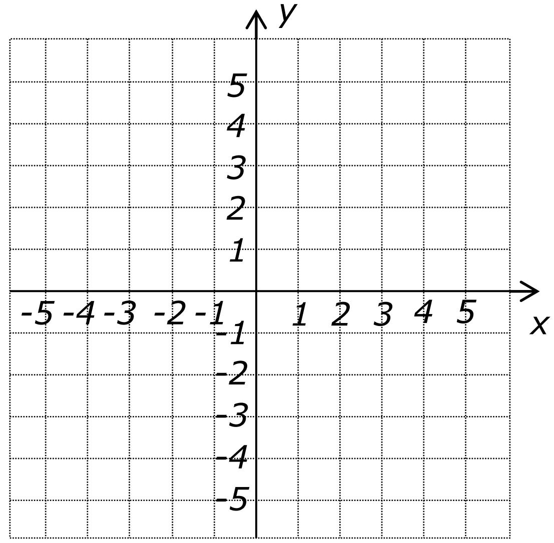
The y-intercept (where the line crosses the y-axis) is $(0, -1)$.

Practice – Graphing

Plot the following:

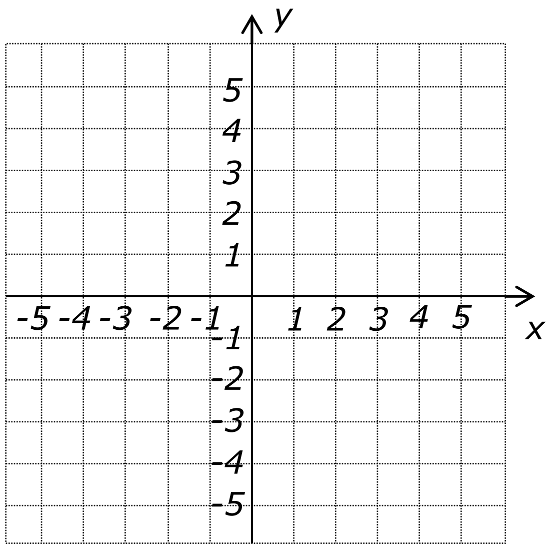
34. (3, −4)

35. (−2, −3)

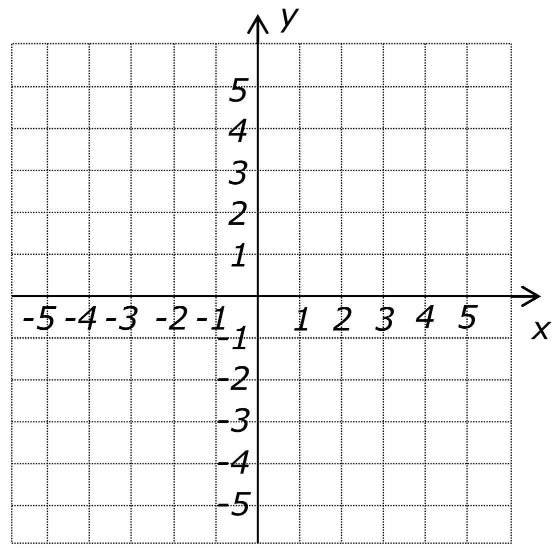


Graph the following linear equations:

36. y = 2x + 3



37. x + 3y = 6

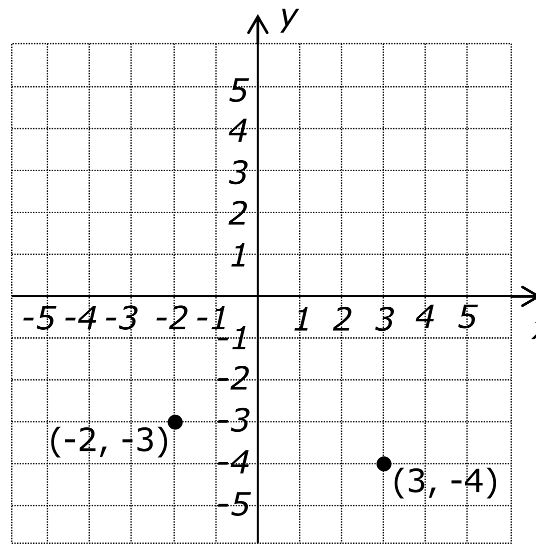


(answers on page 19)

Answers

1. -5
2. -2
3. 6
4. -10
5. 16
6. -4
7. 3.0×10^{6} m/s
8. 2.4×10^{-8} m
9. 56,300,000 tons
10. 0.000 000 002 234 Pa
11. x^5y^3
12. $5xy^2z$
13. 7
14. 2
15. 0.7
16. $-xy^2 + x - 3y^2$
17. $11abc + 18ab$
18. $N = 26$
19. $R = 65$
20. $m = 12$
21. $x = 4$
22. $t = -\frac{4}{3}$
23. $x = 6$
24. 24 cm, 12 cm, & 14 cm
25. $6x^2 + 2x + 2$
26. $4x^3 - 3x^2 - x - 1$
27. $6x^2y^2z^3 + 2x^3z$
28. $6x^4 + 14x^2y - 4x^2z^3 + 4y^2 - 8yz^3$
29. $4w^3 + 3w^2 - 6$
30. $5y(3x^2 - 5y)$
31. $4(x - 2)(x - 2)$ or $4(x - 2)^2$
32. $(5z - 4)(5z + 4)$
33. $(x + 4)(x - 3)$

34. & 35.



36. & 37.