Arithmetic Review Booklet

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The goal of this booklet is not to teach you how to do a particular math problem, but to refresh your memory on skills you once knew but may not have practiced for a while.

Some of this may seem simple, but it is worth practicing the easy stuff so that you are comfortable with the material and how it is presented in this booklet. Feel free to skip sections that you know well.

Times Table

To become good at multiplication, you'll need to know the one-digit multiplication from memory. Even using a calculator, you'll need to know one-digit multiplication to estimate and to check your work.

	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12												

Use addition to fill in the gaps in the table.

For example, 11 x 10 = 110, 11 x 11 = 121 (which is 110 + 11).

To practice one-digit multiplication, complete the table below. Use the table provided to check your work.

x	7	5	9	3	1	8	6	2	4
9									
6				18					
2									
7									
8									
4					4				
5									
1									
3									

Whole Numbers

Estimating answers is an important part of any calculation. If you use a calculator to do the actual arithmetic, it is even more important to first get a reasonable estimate of the answer. If you accidentally hit a wrong key when entering a number, or use a calculator with a failing battery, the calculator may give you a wrong answer. An estimate is your best guarantee that a wrong answer will be caught immediately.

Only do an arithmetic calculation once you know roughly what the answer is going to be. This will also help when doing a multiple choice question since you will know right away which options can be ignored.

Addition

Let's say you need to add 28 and 51. That is roughly 30 + roughly 50 or approximately 80 - not 8 or 800! Having the estimate will keep you from making any major - and embarrassing - mistakes. Once you have a rough estimate, you're ready to do the actual arithmetic work.

(Hint on Rounding: When you round to the nearest ten, round up if the digit in the ones position is 5 or greater. The digit in the tens position remains the same if the digit in the ones position is 4 or less.)

		2	8
	+	5	1
		7	9

Arrange the numbers to be added into columns.

The ones digits are placed on the right in the ones column. The tens digits are placed in the tens column and so on.

Carelessness in lining up the digits is the most common cause of errors in arithmetic.

Add the ones digits → 8 + 1 = 9 Write the answer in the ones place below the numbers being added.

Then, add the tens digits → 2 + 5 = 7 Write the answer in the tens place below the numbers being added.

Our answer, the **sum**, 79, agrees with our original estimate of about 80.

Subtraction

The **difference** is the name given to the answer in a subtraction problem. The difference between 8 and 5 is 3 or 8 – 5 = 3.

What about 48 – 21 = __?

Let's try it without a calculator.

Estimate the answer.

48 – 21 is roughly 50 – roughly 20 so your answer will be roughly 30.

Then write the numbers vertically the same way you did with addition.

Be careful to keep the ones digits lined up in the ones column and the tens digits lined up in the tens column and so on.

$$\begin{array}{cccc} & & 4 & 8 \\ & - & 2 & 1 \\ \hline & & 2 & 7 \end{array}$$

With subtraction, the order the numbers are written is important. The larger number is the one we must write first.

Subtract the ones digits → 8 – 1 = 7 Write your answer below in the ones column.

Subtract the tens digits → 4 – 2 = 2 Write your answer below in the tens columns.

The difference, 27, is roughly equal to our estimate of 30.

Practice - Addition and Subtraction

1.

$$\begin{array}{cccc} & & 9 & 8 \\ & + & 2 & 6 \\ \hline & & & \end{array}$$

2.

$$\begin{array}{cccc} & & 5 & 3 \\ & - & 2 & 8 \\ \hline & & & \end{array}$$

3. 156 + 299 = ______

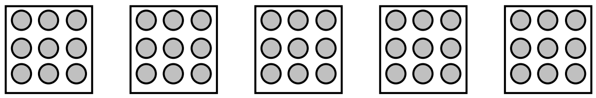
4. 1080 – 536 = ______

(answers on page 23)

Multiplication

Multiplying is a quick way to deal with multiple additions.

For example, if you receive 5 cases of cans and there are 9 cans in each box, you can find the total number of cans by:



Counting: = 45

Adding: 9 + 9 + 9 + 9 + 9 = 45

or multiplying: 5 x 9 = 45

With practice, multiplying is the easiest way to deal with repeated addition. The answer when we multiply is called the **product**.

The two numbers that are being multiplied are often called **factors**. The order of the factors doesn't change the product. 9 x 5 is also 45.

- The product of multiplying a number by one is that number.
- The product of multiplying a number by zero is always zero.
- When multiplying by larger numbers, extend what you know about multiplying one-digit numbers.

Let's try 42 x 3 = ___

First → estimate the answer. 40 x 3 = 120

Second → arrange the factors to be multiplied vertically with the ones digits in the ones column, tens digits in the tens column and so on.

To make the process even more clear, let's write the two-digit number in expanded form:

4 tens + 2 ones
x 3
12 tens + 6 ones = 120 + 6 = 126

Our answer 126 is roughly equal to the estimate of 120.

Division

Division is the reverse of multiplication. Division enables us to separate a given quantity into equal parts. If you want to divide 20 items into 5 equal parts, the question you would ask is "What is 20 divided by 5?"

In symbols, you would write $20 \div 5$, $5\overline{)20}$, $\frac{20}{5}$, or 20/5

All are read "twenty divided by 5"

And, all are entered into your calculator in the same way usually starting with the larger number. If the numbers are not entered in the correct order, the answer won't make sense.

$\boxed{20}$ $\boxed{\div}$ $\boxed{5}$ $\boxed{=}$ 4 This answer makes sense.

The answer when you divide is called the **quotient**.

Having the one-digit multiplication tables firmly in your memory makes division easier since division is like filling in the blanks in multiplication.

For example: $18 \div 3 = \boxed{}$ is really a multiplication: $3 \times \boxed{} = 18$

Those familiar with one-digit multiplying will see that the answer is 6.

Division and zero

Imagine taking something, say 15 pennies, and separating them evenly into zero groups. It's just not possible. You must at least have the original group of one. Division by zero has no meaning and so the answer to any number divided by zero is <u>undefined</u>.

$15 \div 0 \rightarrow$ undefined

This is not the same as dividing zero by something. If you start with nothing, you can divide that evenly into as many groups as you like. Each group gets nothing. So zero divided by any number equals zero.

$0 \div 5 = 0$

Divisions that are more than one-digit multiplications

Sometimes the number you're dividing is the product of more than a one-digit multiplication and so we can't answer from memory. Here is a step-by-step explanation of longer divisions.

Let's try 224 ÷ 7 Estimate → 210 ÷ 7 is roughly 30 (In this case, estimating to the nearest value that will divide evenly by 7.)

$$\begin{array}{r} 32 \\ 7\overline{)224} \\ \underline{-\ 21\downarrow} \\ 14 \\ \underline{-\ 14} \\ 0 \end{array}$$

Step 1 → 7 into 2 doesn't go;
leave a space above the 2;
try 7 into 22.
7 goes into 22 three times.
Write a 3 above the 2.

Step 2 → 3 x 7 = 21. Write 21 below the 22.
Subtract 21 from 22.
Write 1 and bring down the 4.

Step 3 → 2 x 7 = 14. Write 14 below the 14.
Subtract 14 from 14.
Remainder is zero

Check → 32 is roughly equal to the estimate of 30.
Double check by multiplying
32 x 7 = 224

* Note: Division questions do not always come out evenly. There are three ways to write the 'leftovers' when dividing.

1. Write the remainder separately – ie. 21 R5
2. Write the remainder as a fraction – ie. $21\frac{5}{8}$
3. Write the remainder as a decimal – ie. 21.625

Practice – Multiplication and Division

5. $\begin{array}{r} 24 \\ \underline{\times\ 98} \end{array}$

6. $5\overline{)7465}$

7. 47 x 639

8. 5301 ÷ 18

(answers on page 23)

Order of Operations

To prevent confusion and to ensure that we all get the same answer when we perform the same mathematical calculations, mathematicians have established certain rules.

Rule 1 – First, perform any calculations inside brackets (or parentheses).
Rule 2 – Next, perform all multiplications and divisions working from left to right.
Rule 3 – Finally, perform all subtractions and additions working from left to right.

These rules are often summarized and made easier to remember with the acronym: **BEDMAS**.

1. Perform all calculations **B**rackets.
2. Evaluate any **E**xponents.
3. Perform **D**ivision and **M**ultiplication in order.
4. Perform **A**ddition and **S**ubtraction in order.

Don't be afraid to use lots of paper when working out problems involving several calculations. By completely writing out a correct statement each time, you'll be less likely to make mistakes.

Try this one:

$12 \times \underbrace{(4 + 2)} \div 3 - 7$	Perform the calculation within brackets
$\underbrace{12 \times 6} \div 3 - 7$	Divide and Multiply in order from left to right
$\underbrace{72 \div 3} - 7$	
$\underbrace{24 - 7}$	Add and Subtract in order from left to right
17	

Parentheses, brackets and other grouping symbols are used in calculations to signal a departure from the normal order of operations or to make the transition from a written problem clear.

When division problems are given in terms of fractions, brackets are implied but not shown.

$\frac{48 + 704}{117 - 23}$ is equivalent to (48 + 704) ÷ (117 - 23)

The large fraction bar acts as a grouping symbol. The operations in the top and the bottom of the fraction must be completed before the division. Here is how one way to would enter it on a calculator.

$\boxed{(}$ $\boxed{48}$ $\boxed{+}$ $\boxed{704}$ $\boxed{)}$ $\boxed{\div}$ $\boxed{(}$ $\boxed{117}$ $\boxed{-}$ $\boxed{23}$ $\boxed{)}$ $\boxed{=}$ 8

Practice – Order of Operations

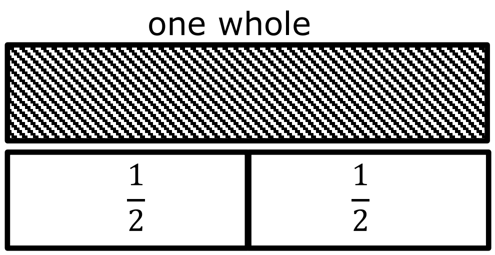
9. $20 + 10 \div 5$

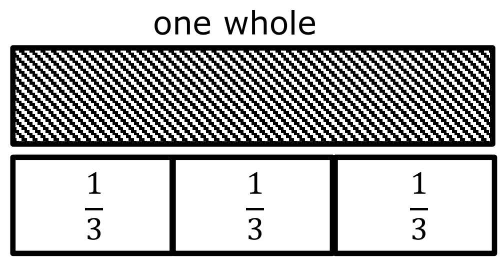
10. $7^2 \times 2 - (12 + 6) \div 3$

11. $9 \times 4 - [(20 + 4) \div 8 - (6 - 4)]$

12. $\frac{75 - 25}{6 + 4}$

Fractions



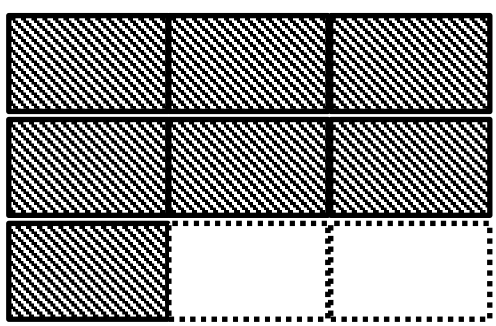


$\frac{1}{2} + \frac{1}{2} = \frac{2}{2}$ = 1 (one whole)

$\frac{1}{3} + \frac{1}{3} + \frac{1}{3} = \frac{3}{3}$ = 1 (one whole)

$$\frac{numerator}{denominator} \qquad \frac{2}{5} \qquad \frac{2\ pieces\ of\ the\ whole}{whole\ is\ cut\ into\ 5}$$

When you see a mixed number such as $2\ \frac{1}{3}$, there are two whole things and $\frac{1}{3}$ of another. We can write this in fractional notation, $\frac{7}{3}$



$$2\frac{1}{3} = \frac{7}{3}$$

To convert a mixed number to a fraction, multiply the whole number by the denominator (2 x 3 = 6) and then add the numerator (6 + 1 = 7).

Write the total over the same denominator (3) to obtain $\frac{7}{3}$.

Multiplication of Fractions

When multiplying fractions there is no need for a common denominator. Simply multiply the numerators and then multiply the denominator. Check the final fraction to see if it can be reduced.

$$\frac{2}{5} \times \frac{3}{4} = \frac{2 \times 3}{5 \times 4} = \frac{6}{20} \text{ which reduces } \frac{6 \div 2}{20 \div 2} = \frac{3}{10}$$

Convert mixed numbers into fractions before multiplying.

$$2\frac{1}{2} \times 3 = \frac{5}{2} \times \frac{3}{1} = \frac{5 \times 3}{2 \times 1} = \frac{15}{2} = 7\frac{1}{2}$$

Division of Fractions

We don’t actually divide fractions. Instead, we ‘flip’ the second fraction and multiply.

$$\frac{2}{5} \div \frac{2}{3} = \frac{2}{5} \times \frac{3}{2} = \frac{2 \times 3}{5 \times 2} = \frac{6}{10} = \frac{3}{5}$$

Convert mixed numbers into fractions before dividing.

$$2\frac{1}{2} \div 3 = \frac{5}{2} \div \frac{3}{1} = \frac{5}{2} \times \frac{1}{3} = \frac{5 \times 1}{2 \times 3} = \frac{5}{6}$$

Practice – Multiplying and Dividing Fractions

13. $\frac{3}{5} \times \frac{5}{8}$

14. $3\frac{1}{3} \times 5\frac{1}{2}$

15. $\frac{1}{3} \div \frac{4}{5}$

16. $10 \div 5\frac{1}{3}$

Addition & Subtraction of Fractions

When you add or subtract fractions, you need to have a common denominator.

$$\frac{1}{5} + \frac{3}{5} = \frac{4}{5}$$

or

$$\frac{4}{7} - \frac{1}{7} = \frac{3}{7}$$

There a several ways to obtain a common denominator. Using multiples is one way. Use whichever method you are most familiar with.

Let's say we had fractions with denominators of 6 and 8.

$$\frac{1}{6} + \frac{5}{8}$$

To find the lowest common denominator, start with the largest number, 8 and work through its multiples (8, 16, 24, 32, …). Look for the first multiple that is also divisible by 6. In this case, 24 is the lowest common denominator. Rewrite each fraction over the common denominator, 24.

$$\frac{1 \times 4}{6 \times 4} = \frac{4}{24}$$

$$\frac{5 \times 3}{8 \times 3} = \frac{15}{24}$$

Keep the denominator and add the numerators.

$$\frac{4}{24} + \frac{15}{24} = \frac{19}{24}$$

So,

$$\frac{1}{6} + \frac{5}{8} = \frac{19}{24}$$

When adding or subtracting mixed numbers, you may wish to convert them to fractions, or you may choose to work with them as mixed numbers. Either works. Remember though, that if a multiple choice question includes mixed numbers, you should choose the answer which is also a mixed number.

For example, this question

$$2\frac{1}{2} + 3\frac{1}{4}$$

can be done both ways.

As mixed numbers
Write the fraction parts with a common denominator. Add the whole numbers and the fraction parts separately. Recombine the whole number and fraction into a mixed number.

$$2\frac{2}{4} + 3\frac{1}{4} = 2 + 3 + \frac{2}{4} + \frac{1}{4} = 5 + \frac{3}{4} = 5\frac{3}{4}$$

As fractions
Convert each to a fraction. Find the common denominator and multiply top and bottom. Then add the numerators and keep the denominator. Convert the answer back into a mixed number.

$$\frac{5}{2} + \frac{13}{4} = \frac{5 \times 2}{2 \times 2} + \frac{13}{4} = \frac{10}{4} + \frac{13}{4} = \frac{23}{4} = 5\frac{3}{4}$$

Subtraction of mixed numbers is done the same way as addition. That is, as improper fraction or as mixed numbers. However, when using mixed numbers, if the second fraction is larger than the first, you will need to borrow from the whole number and break that 'whole' into the appropriate number of parts.

$$10 - \frac{1}{3} = 9\frac{3}{3} - \frac{1}{3} = 9\frac{2}{3}$$

or as fractions:

$$10 - \frac{1}{3} = \frac{10}{1} - \frac{1}{3} = \frac{10 \times 3}{1 \times 3} - \frac{1}{3} = \frac{30}{3} - \frac{1}{3} = \frac{29}{3} = 9\frac{2}{3}$$

Practice – Addition & Subtraction of Fractions

17. $\frac{1}{5} + \frac{2}{5}$

18. $\frac{5}{7} - \frac{2}{7}$

19. $2\frac{1}{2} - 1\frac{1}{4}$

20. $\frac{1}{8} + 2\frac{1}{6}$

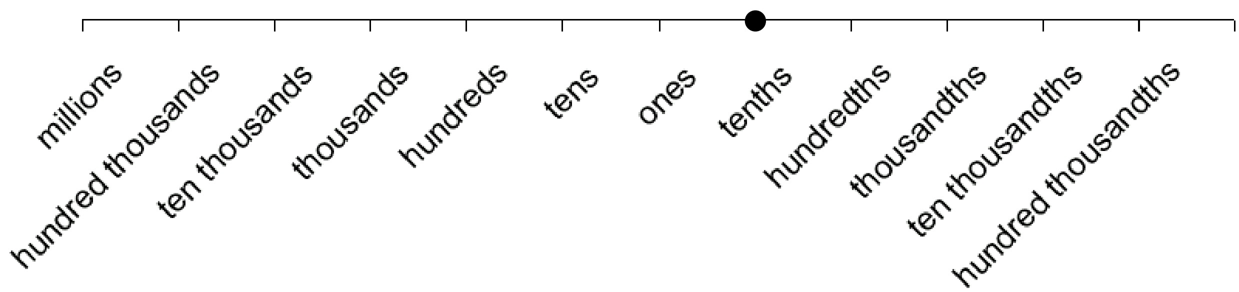
21. $8\frac{1}{4} - 2\frac{7}{8}$

(answers are on page 23)

Decimals

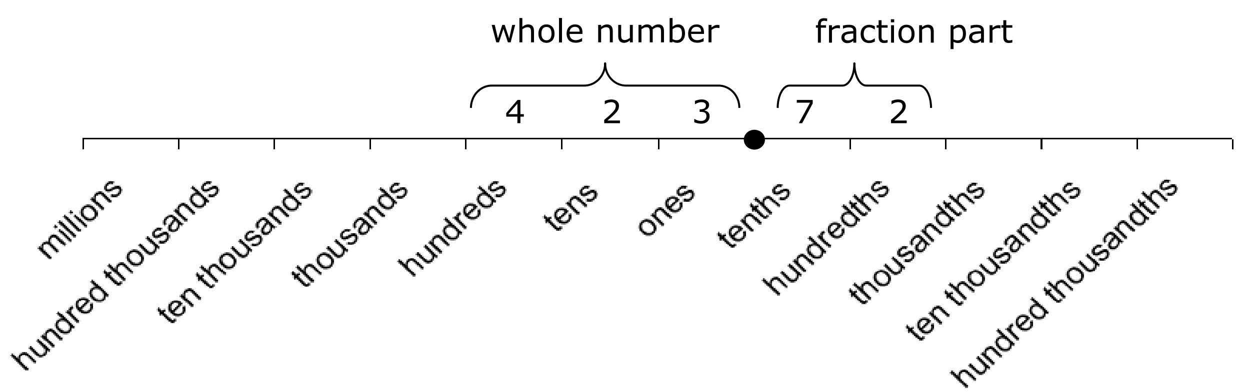
The decimal number system has ten as its base.

The system for whole numbers of keeping each digit in a special place to indicate its value can be extended after the decimal point to show the position of each digit in the fractional part.



A decimal number is a fraction whose denominator is 10 or another multiple of 10.

A decimal number may have both a whole number part and a fraction part. For example, the number 423.72 means:



In words, this number is four hundred twenty-three and seventy-two hundredths.

When converting a decimal number into a fraction, the best way to start is by saying the number out correctly. The position of the last number in the decimal part tells us the denominator for the fraction.

For example, 0.75 is correctly read as seventy-five hundredths, so is written as a fraction $\frac{75}{100}$. This reduces to $\frac{3}{4}$.

Convert fractions to decimals by dividing the numerator by the denominator.

$$\frac{2}{5} = 2 \div 5 = 0.4$$

or

$$\frac{2}{5} = 5\overline{)\,2.0}^{\;0.4} = 0.4$$

Percent is a special kind of decimal fraction that always has a denominator of 100. For example, 90% means $\frac{90}{100}$ or 0.90

Practice – Decimals and Percent

Convert the following into decimal notation:

22. $\frac{3}{8}$

23. $1\frac{1}{4}$

24. 5%

25. 23.4%

Convert the following into fractional notation:

26. 0.2

27. 3.45

28. 50%

29. 9%

(answers are on page 23)

Words for Word Problems

Addition	Subtraction
Sum Plus Add And Total Increase More Raise Both Combine In all Altogether Additional How many?	Difference Less than More than Decrease Reduce Lost Left Remain Fell Dropped Change How much extra? Nearer Further
Multiplication	**Division**
Product Times Total Of Per As much Twice By Area Volume	Quotient Divided (evenly) Split Each Cut Equal pieces Average Every Out of Ratio Shared

Ratio & Proportion

A ratio shows the relationship between two values and often is written much like a fraction. For example, kilometres per hour (km/h) is a ratio of distance travelled to time. A proportion is a way of showing when two ratios are equivalent. In a direct proportion, as one quantity increases, the other increases proportionally. For example, if you travel twice the distance at a constant speed, the time taken will double.

If a car is driven at 50 km/hour, it will travel 100 km in two hours. This can be written as $\frac{50\text{ km}}{1\text{ hour}} = \frac{100\text{ km}}{2\text{ hours}}$

To solve for an unknown in a proportion, we use what is call the cross-products rule. That is, multiplying the quantities that are diagonally opposite, give us the same result.

$$\frac{50}{1} = \frac{100}{2} \qquad 50 \times 2 = 1 \times 100$$

We can use proportions to solve the following problem.

Priscilla bought 3 tickets to a baseball game for \$19.50. At the same price per ticket, how much would she pay for 5 tickets?

Set it up as a proportion, keeping related units together.

$$\frac{5\text{ tickets}}{3\text{ tickets}} = \frac{?}{\$19.50} \quad \text{or} \quad \frac{3\text{ tickets}}{\$19.50} = \frac{5\text{ tickets}}{?}$$

Once it is set up as a proportion, the unknown can be found by multiplying diagonally across the equal sign and dividing by the number which is diagonally across from the unknown.

In this case, 5 × 19.50 ÷ 3 = 32.50

It would cost \$32.50 for 5 tickets.

Practice – Ratio & Proportion

30. $\dfrac{\text{1 part oil}}{\text{20 parts gas}} = \dfrac{?}{\text{5 litres gas}}$

31. Lucy is preparing for a feast for 125 people. She expects that each family of three will eat 20 meatballs. How many meatballs should she make?

32. The ratio of students to teachers on a field trip must be 7 to 2. In a class of 28 students, how many teachers are needed?

33. A team of dogs travels 54 km in 3 hours. Travelling at the same speed, how long will it take them to go 90 km?

Choosing a Calculator

The calculator will help you perform calculations faster, but it will not tell you what to do or how to do it.

If you doubt the calculator, put in a problem whose answer you know, preferably a problem similar to the one you're solving.

Knowing your calculator is extremely important. There are two basic types of calculators. Some scientific calculators use Reverse Polish Logic. This means that you enter the number and then press the key for the function you wish applied. Newer calculators more often use Direct Algebraic Logic. This means that you can enter a formula in exactly the same order as it is written on the page.

To see what kind of calculator you have, try this simple test.

Enter 30 and press the SIN button. If 0.5 displays as the answer, you have a calculator that uses Reverse Polish Logic. You will need to enter the number and then press the button for the function you want applied to that number.

If 0 displays as the answer, or if you got an error message, try pressing the SIN button and then entering 30. Press the Equal button and 0.5 will display. You have a calculator that uses Direct Algebraic Logic (DAL). You will need to enter formulas with functions in exactly the order they are written.

If you're shopping for a calculator, the features to look for include a display of 10 digits, at least one memory (look for keys marked $\boxed{\text{X->M}}$, $\boxed{\text{STO}}$, or $\boxed{\text{M+}}$), keys for calculating square root $\boxed{\surd}$, powers $\boxed{x^2}$ or $\boxed{y^x}$, the three trigonometric functions $\boxed{\sin}$, $\boxed{\cos}$, $\boxed{\tan}$, as well as the four basic arithmetic functions $\boxed{+}$, $\boxed{-}$, $\boxed{\times}$, $\boxed{\div}$

Choosing a calculator with Direct Algebraic Logic (DAL) will make entering long equations and equations needing formulas much simpler.

Answers

x	7	5	9	3	1	8	6	2	4
9	63	45	81	27	9	72	54	18	36
6	42	30	54	18	6	48	36	12	24
2	14	10	18	6	2	16	12	4	8
7	49	35	63	21	7	56	42	14	28
8	56	40	72	24	8	64	48	16	32
4	28	20	36	12	4	32	24	8	16
5	35	25	45	15	5	40	30	10	20
1	7	5	9	3	1	8	6	2	4
3	21	15	27	9	3	24	18	6	12

1. 124
2. 25
3. 455
4. 544
5. 2 352
6. 1493
7. 30,033
8. 294 R9

   Or 294.5

   Or $294\frac{1}{2}$
9. 22
10. 92
11. 35
12. 5
13. $\frac{3}{8}$
14. $18\frac{1}{3}$
15. $\frac{5}{12}$
16. $1\frac{7}{8}$
17. $\frac{3}{5}$
18. $\frac{3}{7}$
19. $1\frac{1}{4}$
20. $2\frac{7}{24}$
21. $5\ \frac{3}{8}$
22. 0.375
23. 1.25
24. 0.05
25. 0.234
26. $\frac{1}{5}$
27. $3\frac{9}{20}$
28. $\frac{1}{2}$
29. $\frac{9}{100}$
30. $\frac{1}{4}$ litre, 0.25 L or 250 mL
31. 834 meatballs
32. 8 teachers
33. 5 hours